Elastic-Plastic Stress Waves Propagation in One-Dimensional Members Using the Method of Characteristics

H. H. Al-Da'ami, A. M. Hosain and N. H. Mostafa

الخلاصة

يتناول هذا البحث التحليل الديناميكي المرن–اللدن للأذرع المستقيمة الموشورية و اللاموشورية و ذلك من خلال دراسة الأمواج المحورية و الأنثنائية المنتقلة عبر هذه التراكيب، و ذلك بأستخدام طريقة الخواص (Method of characteristics). أعتمد البحث على أساس تقريب إنتقال الموجة في تركيب أحادي البعد. نظرية تيموشينكو (Timoshenko's theory)، و المتضمنة تأثير التشوه القصيّ المستعرض و عزم القصور ألدوراني.

لقد وجد إن التداخل بين الموجات القصيّة و الانحنائية يتسبب في إنتقال الموجات الأنثنائية خلال الأذرع الموشورية و اللاموشورية بشكل متشتت ناتج عن الاختلاف في سُرع انتقال تلك الموجات. إن ظاهرة التشتت سوف تزداد عند انتقال الموجات اللدنة خلال تلك الأذرع نتيجة الحركة المعقدة للسطح البيني المرن–اللدن بالإضافة إلى تغير هندسية الذراع. إن هذا التأثير ناتج عن تغير خواص المقطع و المادة على طول العتبة. كما أن تحليل التشوه المحدود (small deformation analysis) لا يعطي نتائج مقنعة عند ظروف متوقعة عالية التحميل و ذلك لأنها تهمل تأثير لا خطية الشكل الهندسي (geometrical non-linearity).

ABSTRACT

This research deals with the elastic-plastic dynamic analysis of prismatic and non-prismatic straight members by modeling axial and flexural waves transmission along such structures, using the method of characteristics. The analysis is based on the wave propagation in one-dimensional structural element approach. Timoshenko's beam theory, which includes transverse shear deformation and rotatory inertia effects, is adopted in the analysis.

It was found that the interaction between shear and bending waves causes the flexural waves to propagate in prismatic and non-prismatic members in a dispersive manner which is caused by the propagation of waves with different velocities. The dispersive phenomenon will increase when the plastic waves are propagated in these members due to the complex movement of the elastic-plastic interfaces and the changes in member’s geometry. The additional dispersion is caused by the changes in sectional and material properties along the member's length. In addition, the small deformation analysis does not give a reasonable result when high loading conditions are expected because it neglects the effect of geometrical non-linearity.

Keywords: stress waves, elastic-plastic response, characteristics method, geometrical and material non-linearity.

1.INTRODUCTION

Recently, there has been a marked interest in the dynamic characteristics deformation of many engineering applications such as high-speed machinery, airplane structure, tall building…etc. The rapid expansion in the study of wave propagation was prompted, in part, by the necessity to understand the transient history of such structures that are subjected to rapidly applied loads.

The beam is one of the fundamental elements of an engineering structure. It is found in the most structural applications. Moreover, structures like helicopter rotor blades, spacecraft antenna, airplane wings, high-rise building, gun barrels, robot arms and subsystems of more complex structures can be modelled as a beam-like slender member **(Malatkar, 2003)**. Therefore, studying the dynamic response, both elastically and plastically, of this simple structural component under various loading conditions would help in understanding and explaining the behaviour of more complex structures under similar loading.

The main objective of this study is to use the method of characteristics to deal with elastic-plastic dynamic analysis of one-dimensional prismatic and non-prismatic straight members, taking into account that these structures are subjected to transient loading. Where this method needs relatively small computer storage with good solution reliability. Thus, the designers can easily overcome the difficulties in the analysis when their designs are subjected to impact and earthquakes loadings. The effect of geometrical and material's non-linearity (bilinear stress-strain relation) are considered when high loading conditions are expected.

A parametric study is presented in order to determine the effect of several influencing parameters on the behaviour of vibrating beam elements.

2. WAVE PROPAGATION IN ONE-DIMENSIONAL STRUCTURAL ELEMENT

The behaviour of structural elements under impact or impulsive loads is a subject of great interest in the structural dynamic analysis. When forces are applied to an elastic-plastic medium over a very short period, the response should be considered in terms of wave propagation theory. The study of transient waves has important implications and applications for structures subjected to such loads. The revival of interest in elastic-plastic wave propagation during the last four decades has been possible because of the rapid development of computing facilities and experimental equipments **(Al-Mousawi, 1983)**.

Waves in one-dimensional structural members can be classified into three groups namely; axial, torsional and flexural waves. The problems of flexural wave propagation have not been so extensively treated as have the problems of axial wave. This is due to the complexities involved in the propagation of flexural waves and their dispersive characters. The Euler-Bernoulli's beam theory is inadequate for studying the transient bending waves because it leads to the physically impossible conclusion that disturbances are propagated instantaneously; the theory also neglects the effects of shear deformation and rotatory inertia **(Al-Mousawi, et al., 1988)**.

Timoshenko's beam theory is the only approximate theory that contains the essential features of exact theory in simplified form **(Achenbach, 1973)**. This theory leads to more accurate solutions than the Euler-Bernoulli's beam theory because the effects of shear deformation and rotatory inertia are included in the governing equations.

3. THE METHOD OF CHARACTERISTICS

The method of characteristics **(MOC)** is used for the numerical solution of first order and second order partial differential equations (PDE) of hyperbolic type. For the hyperbolic PDE, there exist two distinct families of real characteristic curves at each point (*x*,*t*) as shown in Fig.(1). For the parabolic case, the two characteristics coincide and they are of no significant value in understanding the behaviour of the solution, whereas the elliptic form of PDE has no real characteristics [**(Nowacki, 1978),(Mostafa,2005)]**.

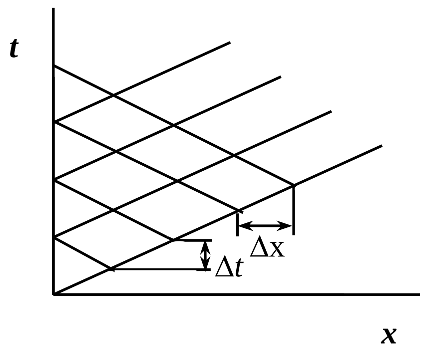

Fig. (1): Characteristic network.

The real characteristics of hyperbolic PDE are curves in the real domain of the problem, and discontinuities will propagate along the characteristics. An explicit step-by-step process is usually used in building up simultaneously the characteristic grid and solving the PDE at the grid points.

The main advantage of the method of characteristics is that discontinuities in the initial values may propagate along the characteristics. Furthermore, the method of characteristics has the advantage that it follows the physical waves fronts as they are propagated along the beam **(Al-Mousawi, et al., 1988)**. Thus, the method of characteristics is used in this study to overcome the disadvantages of other techniques.

4. THEORETICAL ANALYSIS

(A) Axial stress Waves

When the elastic dynamic load is subjected to an element over a very short period, the response should be considered in terms of elastic wave propagation theory. Here, the deformation caused by this load will recover when the load is removed.

Fig. (2) shows a general element with length *dx* under axial loading condition.

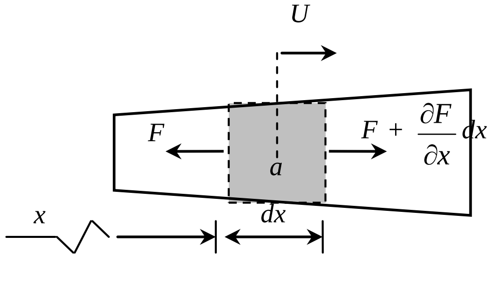

Fig.(2): Axial motion of a general element

The characteristics equations of such element for the elastic vibration are **(Mostafa, 2005)**:

$$\frac{dF}{dt} \pm \rho \ a \ C_e \ \frac{dU}{dt} = 0 \qquad (1)$$

where C_e is the elastic wave's speed that equal to:

$$C_e = \sqrt{\frac{E}{\rho}} \qquad (2)$$

and for the plastic loading wave, the characteristics equations will become:

$$\frac{dF}{dt} \pm \rho \ a \ C_p \ \frac{dU}{dt} = 0 \qquad (3)$$

where C_p is the plastic wave's speed that equal to:

$$C_P = \sqrt{\frac{E_1}{\rho}} \qquad (4)$$

Furthermore, for the unloading axial wave, the characteristics equations has the form **(Cristescu, 1967)**:

$$\frac{dF}{dt} \pm \rho \ a \ C_e \ \frac{dU}{dt} = a \ C_e \ \frac{d\sigma_m}{dx} - a \ E \ \frac{dU_m}{dx} \qquad (5)$$

In addition, the new grid length and element's area are equal to:

$$\Delta S = (1 + \varepsilon) \Delta x \qquad (6)$$

$$a_{(t+\Delta t)} = \frac{a_{(t)}}{(1+\varepsilon)} \qquad (7)$$

(B) Flexural Stress Waves

The problem of flexural stress wave propagation in beams has not been so extensively treated as has the problem of axial wave propagation. This is due to the complexities involved in the propagation of such waves and their dispersive character **(Al-Mousawi, et al., 1988)**.

Timoshenko's beam theory **(1974)** is considered to be only approximated theory that contains the essential features of the exact theory in a simplified form. This theory leads to more accurate solutions than the Euler-Bernoulli's theory because the effects of shear deformation and rotatory inertia are included in the governing equations. In this theory, to simplify the derivation of the equations of motion, the shear stress is assumed to be uniform over a given cross section. In turn, the shear correction factor is introduced into account for this simplification, and its value depends on the shape of the cross section, **[Cowper (1966)**, **Gruttmann** and **Wanger (2001)]**.

Now consider the small transverse vibrations of a general element as shown in **Fig.** (3).

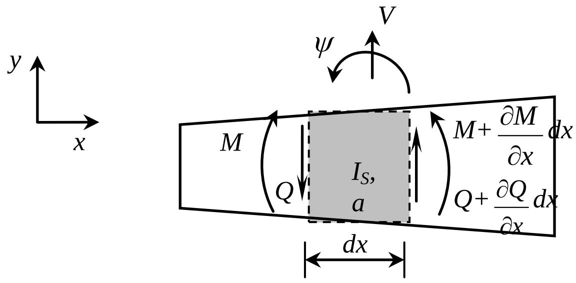

Fig. (3): general element under flexural vibration (elastic)

The characteristics equations of such element for the elastic vibration are**(Mostafa, 2005)**:
For shear,

$$\frac{dQ}{dt} \pm \rho\ a\ C_{es}\ \frac{dV}{dt} = -K\ a\ G\ \psi \tag{8}$$

For bending moment,

$$\frac{dM}{dt} \pm \rho\ I_S\ C_e\ \frac{d\psi}{dt} = \pm\ C_e\ Q \tag{9}$$

where

$$C_{es} = \sqrt{\frac{KG}{\rho}} \tag{10}$$

and the characteristics equations for the plastic vibration ,according to **Fig**. **(4)**, are**(Mostafa, 2005)**:

For axial,

$$\frac{d}{dt}[\ F\ cos(\ \Psi\ )] - \frac{d}{dt}[\ Q\ sin(\ \Psi\ )] \pm \rho\ a\ C_p\ \frac{dU}{dt} = 0 \tag{11}$$

For shear,

$$\frac{d}{dt}[\ F\ sin(\ \Psi\ )] + \frac{d}{dt}[\ Q\ cos(\ \Psi\ )] \pm \rho\ a\ C_{ps}\ \frac{dV}{dt} = -K\ a\ G_1\ \psi \tag{12}$$

For bending moment,

$$\frac{dM}{dt} \pm \rho\ I_s\ C_p\ \frac{d\psi}{dt} = \pm\ C_p\ Q \tag{13}$$

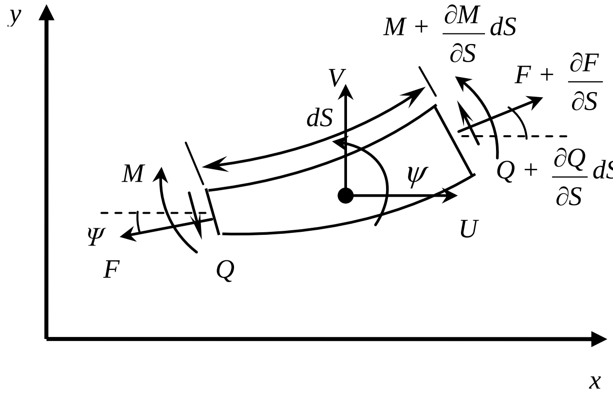

Fig. (4): general element undergoing flexural vibration (plastic)

Where

$$C_{ps} = \sqrt{\frac{KG_1}{\rho}} \tag{14}$$

The displacement components *u* & *v* will be adopted as the two basic parameters in defining other geometrical quantities. Consider the typical element whose original axial length before deformation is *dx* as depicted in **Fig**. (4). After deformation, its length becomes *dS*, which is related to *dx* by **(Reid et al, 1998)**,

$$dS = \sqrt{(\frac{dv}{dx})^2 + (1+\frac{du}{dx})^2}\ dx \tag{15}$$

The quantities Ψ, $sin(\Psi)$, $cos(\Psi)$ in eqs. (11)& (12) are expressed in terms of *u & v* by **(Reid et al, 1998)**,

$$\Psi = tan^{-1}\left(\frac{dv/dx}{1+du/dx}\right) \tag{16}$$

$$sin(\Psi) = \frac{dv/dx}{\sqrt{(dv/dx)^2 + (1+du/dx)^2}} \tag{17}$$

$$cos(\Psi) = \frac{1+du/dx}{\sqrt{(dv/dx)^2 + (1+du/dx)^2}} \tag{18}$$

And the curvature is equal to:

$$k = \frac{d\Psi}{dS} = \frac{(1+du/dx)\,d^2v/dx^2 - (dv/dx)(d^2u/dx^2)}{[(dv/dx)^2 + (1+du/dx)^2]^{1.5}} \tag{19}$$

Furthermore, for the unloading flexural wave, the characteristics equations has the form**(Mostafa, 2005)**:

For axial,

$$\frac{d}{dt}[\, F \cos(\Psi)] - \frac{d}{dt}[\, Q \sin(\Psi)] \pm \rho\, a\, C_e \frac{dU}{dt} = a\, C_e \frac{d}{dS}\{\, \sigma_m \cos[\Psi_m] - \tau_m \sin[\Psi_m]\} - a\, E\, (\frac{dU_m}{dS}) \tag{20}$$

For shear,

$$\frac{d}{dt}[\, F \sin(\Psi)] + \frac{d}{dt}[\, Q \cos(\Psi)] \pm \rho\, a\, C_{es} \frac{dV}{dt} + K\, a\, G \psi = a\, C_{es} \frac{d}{dS}\{\, \sigma_m(S) \sin[\Psi_m(S)] + \tau_m(S) \cos[\Psi_m(S)]\} - K\, a\, G\, [\frac{dV_m(S)}{dS} - \psi_m(S)] \tag{21}$$

For bending moment,

$$\frac{dM}{dt} \pm \rho\, I_s\, C_e \frac{d\psi}{dt} = \pm\, C_e\, Q + C_e \frac{dM_m}{dS} - E\, I_s \frac{d\psi_m}{dS} \tag{22}$$

5. PARAMETRIC STUDY

A parametric study is performed to assess the influence of several important parameters on the dynamic behaviour of straight members.

The selected parametric studies in this chapter can be summarized as follows:

a. The effect of tapering ratio.

b. The effect of sudden discontinuity in cross section.

c. The effect of several bonded materials.

d. The effect of large deformation behaviour.

e. The effect of plastic modulus.

f. The effect of elastic yield limit.

Each one of the above parameters is studied individually by analyzing the geometry of the member, material properties and type of external excitation.

a. The Effect of Tapering Ratio

To show the effect of tapered ratio on the elastic dynamic response of the member structure, a non-prismatic cantilever beam with rectangular cross section is analyzed. All the details of the beam's properties and the dynamic characteristics are given in **Fig. 5**.

Fig. 11 shows that when the tapered ratio **(h_1/h_2)** increases from **(1.5)** to **(3)**, the amplitude of the axial force, at the fixed end, will increase and become more dispersive because of increasing the change in the cross section between any two adjacent elements and this leads to more disturbance characteristics caused by the wave reflection.

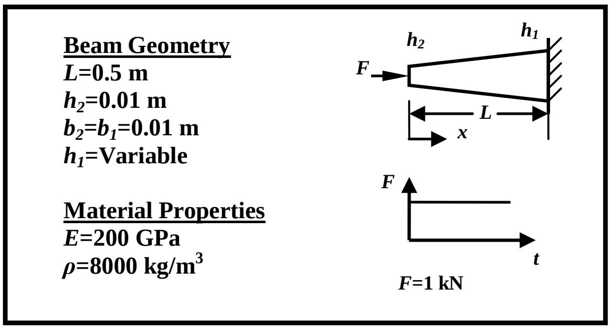

Fig. (5): Details of beam's properties and dynamic characteristics.

b. The Effect of Sudden Discontinuity in Cross Section

To assess the effect of sudden discontinuity in the cross section, a cylindrical cantilever beam made from the same material throughout its length with a discontinuity introduced by the change of the diameter in the mid-span of the beam is analyzed. All the details of the beam's geometry and the dynamic characteristics are given in **Fig. 6**.

Fig. **12** shows the change in the diameter ratio for a steel beam on time history of bending moment at **0.2475 m** from the free end, i.e. before the discontinuity, where the lateral force and bending moment are applied. Four different cases of diameters ratios **DR=1, 1.5, 2, 2.5** are investigated. It is shown that the peak values of the bending moment for the three stepped cases are decreased with decreasing the diameters ratio. This indicates that the reflected bending wave, with the same sign, through the discontinuity increases with increasing the difference between the two diameters.

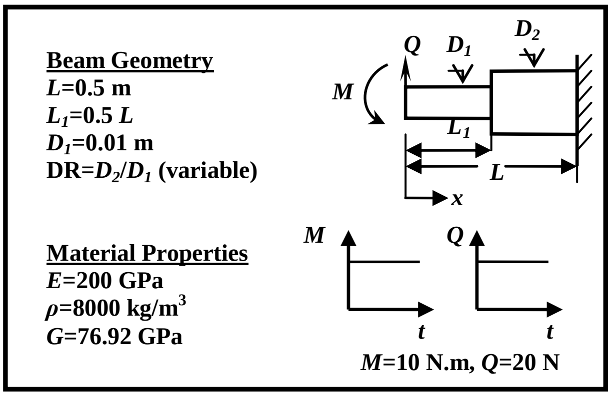

Fig. (6): Details of beam's properties and dynamic characteristics.

c. The Effect of Several Bonded Materials

To show the effect of several bonded materials on the dynamic behaviour of beam, a prismatic cantilever beam, square in cross section, consists of two materials fused together at the mid-span is analyzed. All the details of the beam geometry and the dynamic characteristics are given in **Fig**. **7**.

Fig. **13** shows the reflection of the axial wave in opposite direction when it reaches the position of discontinuity (material's property change). It can be noticed that the magnitude of the reflected wave depends on the difference of the term ρC_e for the two materials on the two sides of the junction point.

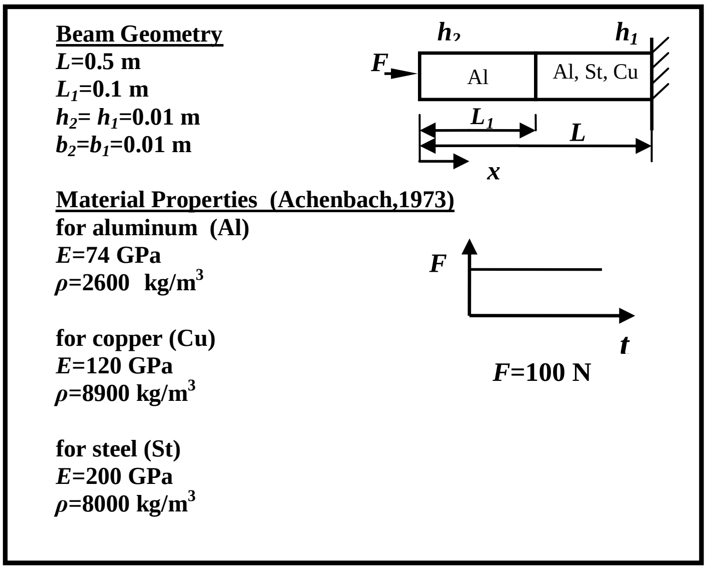

Fig. 7: Details of beam's properties and dynamic characteristics.

d. The Effect of Large Deformation Behavior

To assess the effect of large deformation behaviour, or so-called geometrical non-linearity effect, on the elastic-plastic dynamic response of one-dimensional member, a prismatic cantilever beam with square cross-sectional area is analyzed. All the details of the beam's geometry and the dynamic characteristics are given in **Fig. 8**.

Fig. 14 a, b shows that the effect of longitudinal displacement cannot, however, be ignored for very large deflection because it will give incorrect prediction in the periods and the amplitudes of the vibration. Moreover, it can be noticed that the large deformation analysis gives more dispersive (wavelength is changed as the wave propagates through the medium) effects than the small deformation analysis. This event is caused by the change in geometrical properties, the length and the cross section, of the beam, where the beam becomes no more prismatic.

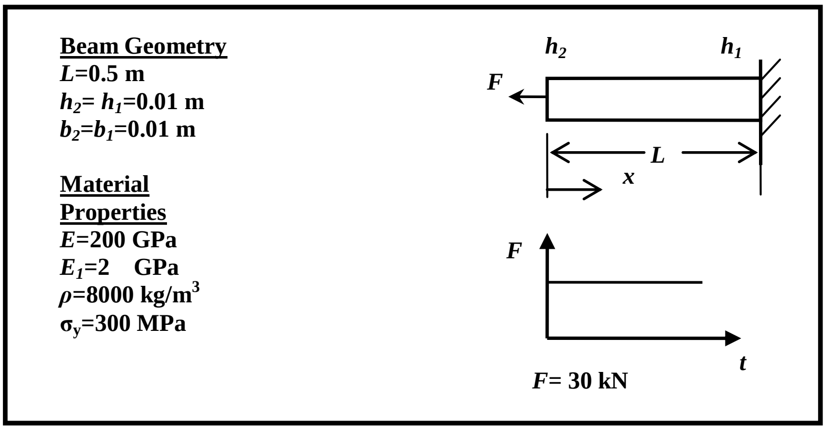

Fig. 8: Details of beam's properties and dynamic characteristics.

e. The Effect of Plastic Modulus

To study the effect of plastic modulus on the elastic-plastic response of vibrating beam, a prismatic cantilever beam with square cross-sectional area is analyzed. All the details of the beam geometry and the dynamic characteristics are given in **Fig. 9**.

Fig. 15 a, b shows that at a time before **0.4 ms** , the elastic waves that propagated with a stress equal to the yielding stress will vibrate at a large amplitude. After that, the plastic waves will reach the mid-span of the beam. At this moment, the kinetic energy will begin to decrease due to local plastic deformations along the beam's length (energy conservation). Moreover, it can be noticed that when the plastic modulus decrease (i.e., the beams becomes more softening), the kinetic energy is also decreased due to increasing the strain energy (accumulative plastic deformation) that stored in the member.

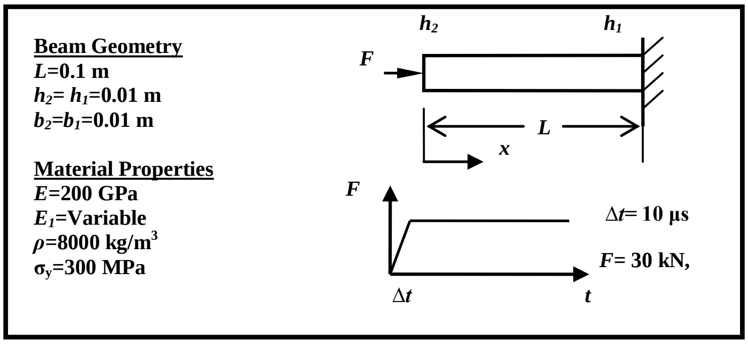

Fig. 9: Details of beam's properties and dynamic characteristics.

f. The Effect of Elastic Yield Limit

To study the effect of the elastic yield limit on the elastic-plastic dynamic behaviour of the beam structure, a non-prismatic cantilever beam with rectangular cross-sectional area is analyzed. All the details of the beam geometry and the dynamic characteristics are given in **Fig. 10**.

Fig. 16 a, b, c, d shows that the onsets of yielding are delayed with increasing the yield limit of the material. Where, the onset of yielding are taken place at times **0.3286**, **0.8073**, and **1.6233 ms** with the corresponding yielding limits equal to **200**, **300**, and **400 MPa** respectively. In addition, it can be noticed that the location of the first yielding is propagated forward the fixed end of the cantilever beam with increasing the yielding limit due to increasing the moment arm, i.e. increasing the bending moment. In addition, it can be shown that there was some disturbance near the positions of the first yielding that caused by the instantaneous change in the material's property (elastic-plastic interface).

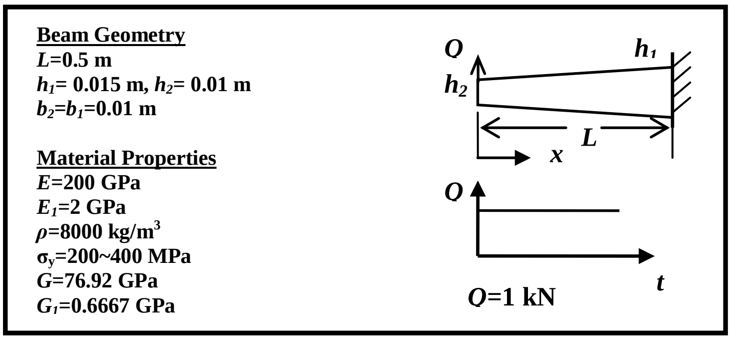

Fig. 10: Details of beam's properties and dynamic characteristics.

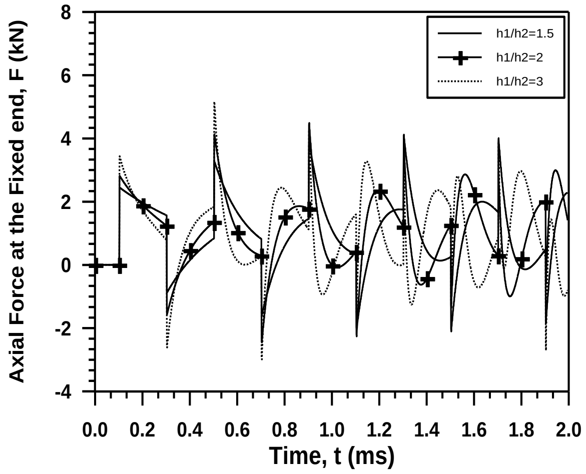

Fig.11

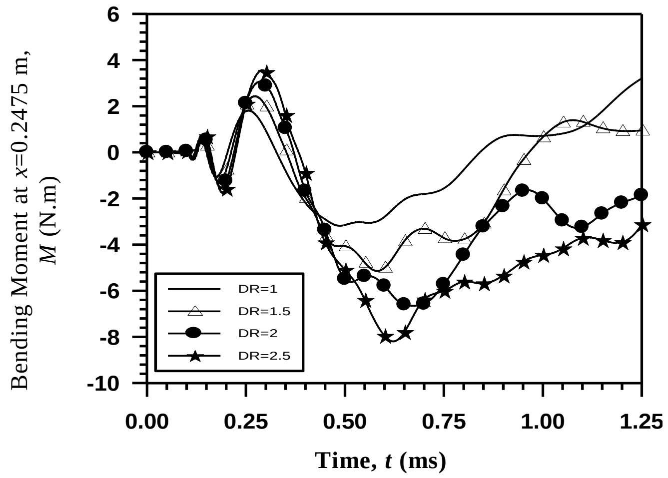

Fig.12

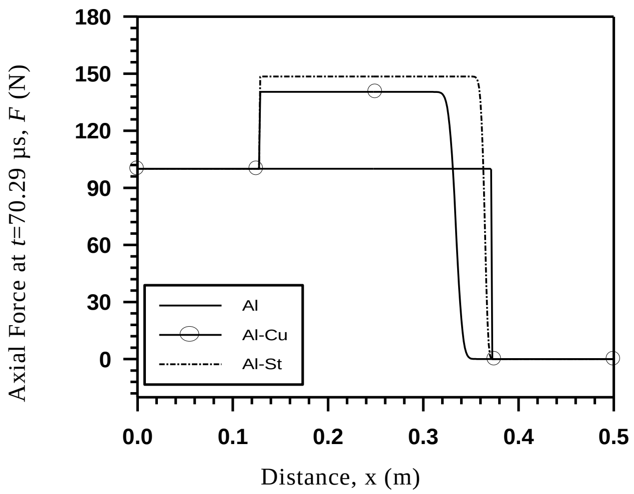

Fig. 13

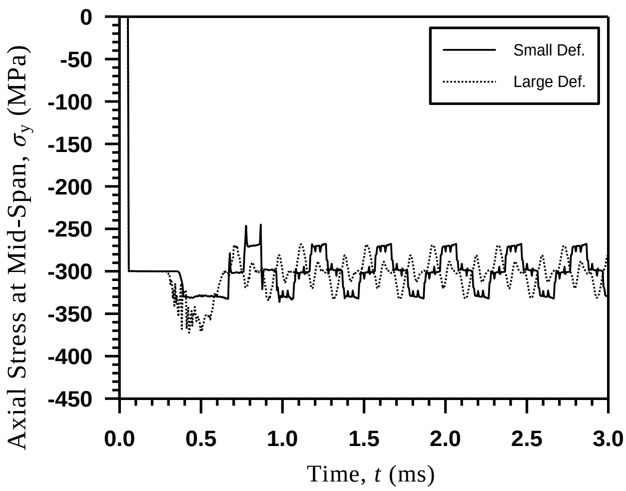

Fig. 14 a

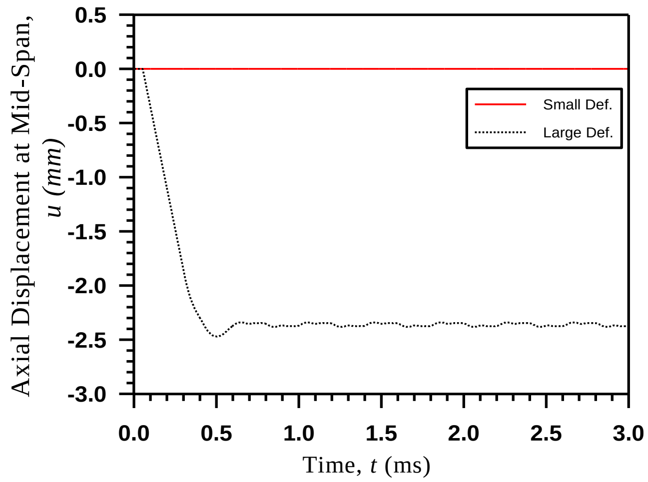

Fig. 14 b

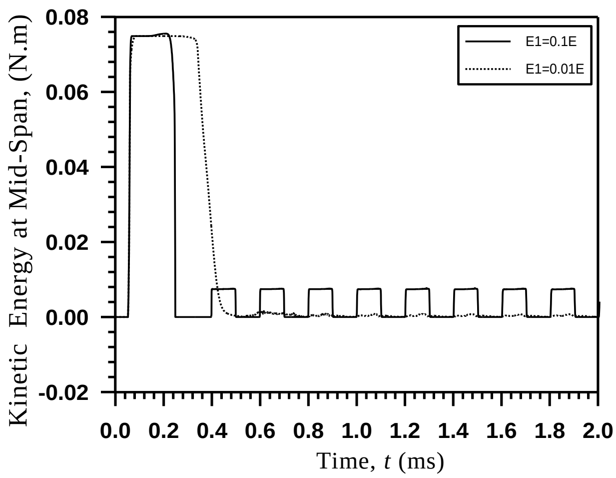

Fig. 15 a

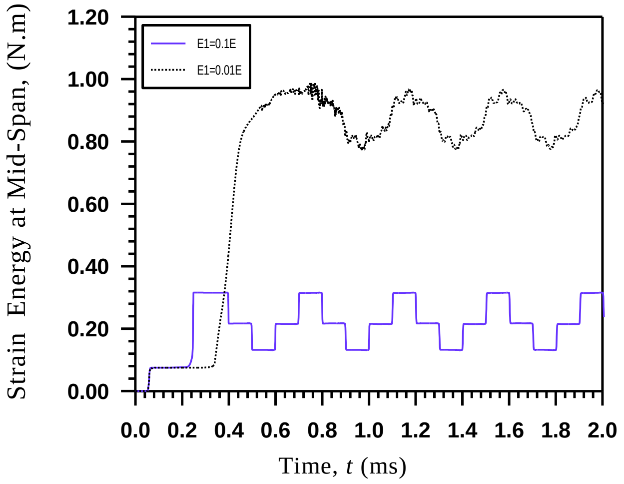

Fig. 15 b

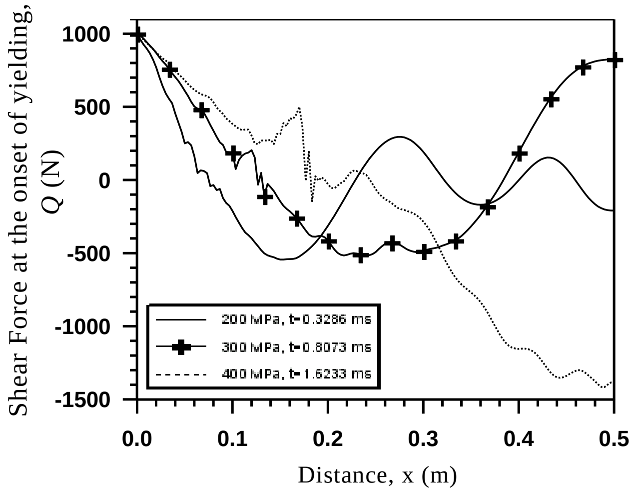

Fig. 16 a

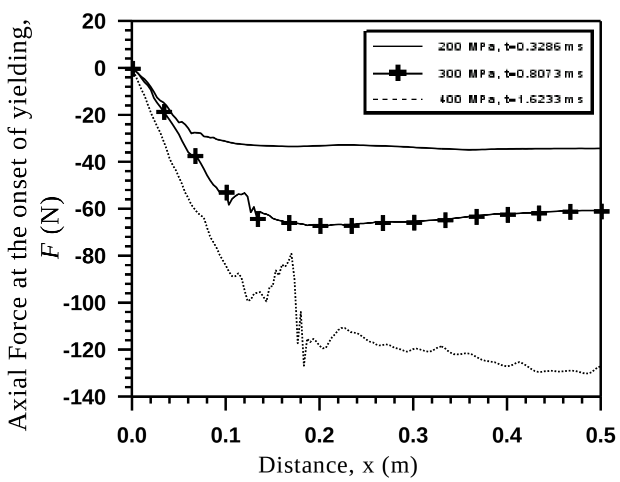

Fig. 16 b

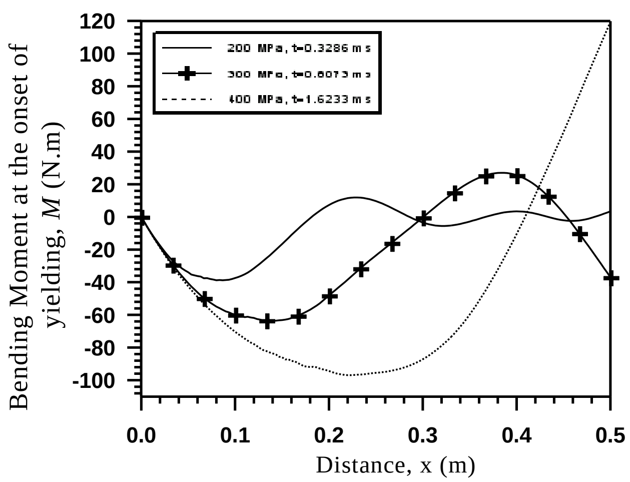

Fig. 16 c

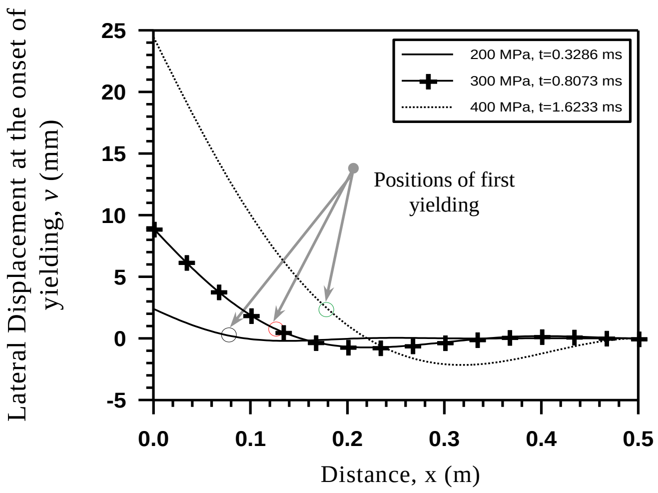

Fig. 16 d

7. CONCLUSIONS

Based on the results obtained from the present analysis procedure, the following conclusions can be drawn:

1. The interaction between shear and bending waves causes the flexural waves to propagate in prismatic and non- prismatic members in a dispersive form.
2. The method can efficiently predict forces, velocities and displacements throughout a structure under consideration at any instant.
3. Non-linear behaviour should be adopted when high loading conditions are expected.
4. In large deformation analysis, flexural waves will generate axial waves as a result of geometric non-linearity.
5. Discontinuities of cross-sectional area and the change in material properties along the longitudinal axis of the member have important effects on the amounts of reflection and transmission stress waves and their oscillations.

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NOTATIONS

Symbol	*Definition*	*Units*
A	*Cross-sectional area at node*	m^2
a	*Average cross-sectional area of the element*	m^2
b	*Width of the element*	m
C_e	*Elastic axial and moment wave speeds*	m/s^2
C_{es}	*Elastic shear wave speed*	m/s^2
C_p	*Plastic axial and moment wave speeds*	m/s^2
C_{ps}	*Plastic shear wave speed*	m/s^2
E	*Modulus of elasticity*	N/m^2

E_1	*Plastic modulus*	N/m^2
F	*Axial force*	N
G	*Elastic modulus of rigidity*	N/m^2
G_1	*Plastic modulus of rigidity*	N/m^2
h	*Depth of the element*	m
I	*Moment of inertia at node*	m^4
I_s	*Average moment of inertia of the element*	m^4
K	*Shear correction factor*	-
k	*Bending curvature*	m^{-1}
L	*Length of the member*	m
M	*Bending moment*	$N.m$
M_m	*Maximum bending moment*	$N.m$
Q	*Shear force*	N
t	*Time*	s
U	*Axial velocity*	m/s
U_m	*Maximum axial velocity*	m/s
u	*Axial displacement*	m
V	*Lateral velocity*	m/s
V_m	*Maximum lateral velocity*	m/s
v	*Lateral displacement*	m
x,y	*Cartesian axes*	m
ΔS	*Grid length after deformation*	m
Δt	*Time incremental*	s
Ψ	*Angular displacement*	rad
Ψ_m	*Maximum angular displacement*	rad
ψ	*Angular velocity*	rad/s
Ψ_m	*Maximum angular velocity*	rad/s
ρ	*Mass density*	Kg/m^3
ε	*Axial strain*	-
ε_m	*Maximum axial strain*	-
σ	*Axial stress*	N/m^2
σ_m	*Maximum axial stress*	N/m^2
σ_y	*Elastic yield limit*	N/m^2
τ	*Shear stress*	N/m^2
τ_m	*Maximum shear stress*	N/m^2
γ	*Shear strain*	-
γ_m	*Maximum shear strain*	-